A Provably Convergent Heuristic for Bicriteria Stochastic Integer Programming

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1 Extended Abstract

The area of combinatorial optimization has been enlarged in the recent years into two directions: First, a huge number of articles deals with *multiobjective combinatorial optimization* (MOCO) problems, for which techniques to determine the set of Pareto-optimal solutions have been developed (cf., e.g., [6], [3]). A major advantage of MOCO approaches is that they are able to provide the decision maker with a small set of reasonable decision alternatives, leaving to her/him the final choice on a management level, but dispensing her/him from the charge of considering, evaluating and comparing thousands or millions of possible decision options from which only a very small fraction finally turns out as promising. In this way, the MOCO decision analysis paradigm can contribute in a substantial way to the development of modern decision support systems. Especially the solution of MO problems by metaheuristic techniques has recently attracted the attention of many researchers [3].

Secondly, *stochastic combinatorial optimization* (SCO) problems, representing uncertainty on problem parameters by means of a stochastic model, have led to the development of a further class of techniques which apply, before or during optimization, numerical procedures or simulation for obtaining characteristic values of distributions from the underlying stochastic model (cf., e.g., [2]). Also for SCO, metaheuristic approaches find increasing interest. In the metaheuristic community, SCO is often termed noisy combinatorial optimization; a recent survey can be found in [5].

The large body of literature in both the MOCO and the SCO field could lead to the conjecture that there might also be a considerable intersection domain between both fields, i.e., articles combining multiobjective and stochastic features in combinatorial optimization. Indeed, from the perspective of various applications (e.g., in vehicle routing, project scheduling, software engineering, or health care management), it would be very desirable to be able to cope with problems that are both of a multicriteria type *and* incorporate uncertainty. Surprisingly, however, literature in this intersection, which we call *multiobjective stochastic combinatorial optimization* (MOSCO), is scarce, as has also been noted in [2]. In [4], two general-purpose metaheuristic solution algorithms SP-ACO and SP-SA determining approximations to the set of Pareto-optimal solutions for instances from a large class of MOSCO problems are presented. Some experimental results are given, but the proposed algorithms do not yet come with a convergence guarantee.

The present article tries to overcome the mentioned drawback of [4] at least for the special case of *bicriteria* SCO problems by presenting a solution algorithm that can be shown to converge, on certain conditions, with probability one to the true solution set. Although, in principle, the approach can be extended to arbitrary CO problems, we shall focus here on the case where the underlying deterministic single-objective problem is an *integer linear problem* (ILP). For this case, we use an ILP solver as a subprocedure of the presented algorithm. (In the more general case, the ILP solver would have to be replaced by another solution procedure, also metaheuristic procedures with known convergence properties being a feasible option.) The ILP solver has to be embedded into an algorithmic context where it is able to deal with (deterministic) *bicriteria* ILPs; for this purpose, we apply the algorithm by Chalmet et al. [1].

The key idea of the presented approach is to combine the repeated solution of deterministic bicriteria problems with a sampling approach. If this is done in a suitable way, a current set of proposed solutions can be shown to approach the set of Pareto-optimal solutions, provided that the latter set contains elements that are sufficiently separated from each other in the objective space, and that the underlying noise distribution is Gaussian. (The last assumption may seem rather strong, but we will argue that this is not the case.) It should be noted that the proposed approach is not only able to find *supported* solutions, i.e., solutions of the bicriteria problem that are optimal with respect to weighted-average problems with certain weights, but that it also identifies non-supported Pareto-optimal solutions of the bicriteria problem.

The full paper will have the following content: First, a precise problem formulation and the proposed solution algorithm are presented. Then, the mentioned mathematical convergence result is derived and discussed. Finally, an experimental illustration of the approach by applying it to a bicriteria stochastic knapsack problem, using an XPRESS implementation, is given.

References

[1] Chalmet, L.G., Lemonidis, L., Elzinga, D.J., "An algorithm for the bi-criterion integer programming problem", *European J. of Operational Research* 25, pp. 292-300 (1986).

[2] Fu, M.C., "Optimization for simulation: theory vs. practice", *INFORMS J. on Computing* 14, pp. 192-215 (2002).

[3] Gandibleu, X., Sevaux, M., Sörensen, K., T'kindt, V. (Eds.), *Metaheuristics for Multiobjective Optimization*, Springer, Berlin-Heidelberg (2004).

[4] Gutjahr, W.J., "Two metaheuristics for multiobjective stochastic combinatorial optimization," *Proc. SAGA 2005* (Stochastic Algorithms: Foundations and Applications), eds.: O.B. Lupanov, O.M. Kasim-Zade, A.V. Chaskin, K. Steinhoefl, Springer Lecture Notes in Computer Science 3777, pp. 116–125 (2005).

[5] Jin, Y., Branke, J., "Evolutionary optimization in uncertain environments – a survey", *IEEE Trans. on Evolutionary Computation* 9, pp. 303-317 (2005).

[6] Steward, Th.J., Honert, R.C. van den (Eds.), *Trends in Multicriteria Decision Making*, Springer, Berlin (1998).