Decomposition, relaxation and metaheuristic generation

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Large part of combinatorial optimization research has been devoted to the study of exact methods leading, most notably during the years 60s and 70s, to a number of very diversified approaches to the solution of the problems of interest. Several of these approaches have by now lost their thrust in the literature as newer, more powerful exact optimization techniques appeared and proved their effectiveness. However, some of those older framework can now be revisited in a metaheuristic perspective, as they are quite general frameworks for dealing with optimization problems.
In this work we propose to investigate the possibility of reinterpreting decomposition, with special emphasis on the related Dantzig-Wolfe, Benders and Lagrangean relaxation techniques. We show how these techniques, when applied in a heuristic context, can be framed as a "master process that guides and modifies the operations of subordinate heuristics", i.e., as metaheuristics [3]. Obvious advantages arise from these approaches, first of all the runtime evolution of both upper and lower bounds to the optimal solution cost, thus yielding both an high-quality heuristic solution and a runtime quality certificate of that same solution.
We have implemented two such approaches and validated their effectiveness. We used a Benders based code to solve a Multi-Mode Project Scheduling Problem [2] and a parallel, asynchronous Lagrangean code to solve a problem arising in peer to peer network operations, namely the Membership Overlay Problem [1].

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